Set theory - Winter semester 2016-17

Problems	Prof. Peter Koepke
Series 7	Dr. Philipp Schlicht

Problem 28 (4 points)**.** The *lexicographical ordering* $<_{\text{lex}}$ on Ord × Ord is defined by

$$(i,j) <_{\text{lex}} (k,l) \iff (i<k) \text{ or } (i=k \text{ and } j<l).$$

Prove the following statements for ordinal arithmetic.

(a) $\langle \alpha+\beta, <\rangle$ is isomorphic to $\langle (\{0\}\times\alpha)\cup(\{1\}\times\beta), <_{\text{lex}}\rangle$.

(b) $\langle \alpha\cdot\beta, <\rangle$ is isomorphic to $\langle \beta\times\alpha, <_{\text{lex}}\rangle$.

Problem 29 (6 points)**.** Suppose that κ is an infinite cardinal. Determine the cardinality of the following sets.

(a) The set ${}^{<\omega}\kappa = \bigcup_{n\in\omega} {}^{n}\kappa$ of tuples in κ.

(b) The set of finite subsets of κ.

(c) The set of rational numbers.

(d) The set of functions $f\colon \kappa\to\kappa$.

(e) The set of bijections $f\colon \kappa\to\kappa$.

(f) The set of strictly monotone functions $f\colon \kappa\to\kappa$.

Problem 30 (4 points)**.** Prove that $\text{card}(\mathbb{R}^+) = 2^\omega$.

Problem 31 (4 points)**.** Prove the following statements.

(a) $\prod_{n\in\omega} \aleph_n = (\aleph_\omega)^\omega$.

(b) If the CH holds (i.e. $2^{\aleph_0}=\aleph_1$), then $\aleph_n^\omega = \aleph_n$ for all $n\geq 1$.

Problem 32 (4 points)**.** Prove that $\kappa^{\text{cof}(\kappa)} > \kappa$ for all infinite cardinals κ, without using König's theorem. *(Hint: as in the proof of Cantor's theorem, suppose that there is a list of all functions $f\colon \text{cof}(\kappa)\to\kappa$ and construct a function $g\colon \text{cof}(\kappa)\to\kappa$ which is not in this list.)*

Due Friday, December 9, before the lecture.